

Supplementary Document

Graph-Theoretic Definitions

Definition 1 (Graph). The term graph is used throughout to denote an unweighted and undirected simple graph (without self-loops or parallel edges) $G = (V, E)$, where V and E are the vertex and edge sets, respectively. Here E is represented as a set of vertex-pairs, i.e., $E = \{(u, v) | u, v \in V\}$.

Definition 2 (Degree of a Vertex). The *degree* of a vertex v_i , denoted as $d(v_i)$ in a graph, is said to be the number of edges incident to it. Hence $d(v_i) = |\{(v_i, v_j) \in E, v_j \neq v_i\}|$.

A graph $G = (V, E)$ may contain subgraphs. A clique is a complete subgraph of a graph.

Definition 3 (Clique). A subgraph $G = (V, E)$ is said to be a clique if for each vertex pair $u, v \in V$, there is an edge (u, v) .

As can be seen, the edge set E of a clique can readily be obtained from the vertex set V , and therefore a clique may be simply denoted as $G = V$.

Definition 4 (γ -quasi-clique). In a graph $G = (V, E)$, a subgraph $G = (V', E')$, $V' \subseteq V$, $E' \subseteq E$, is said to be a γ -quasi-clique ($0 \leq \gamma \leq 1$) if the subgraph induced by this set of vertices contains at least $\lceil \gamma \cdot |V'| C_2 \rceil$ edges.

We denote the cardinality of a vertex set V as $|V|$.

A graph is bipartite if its vertex set can be distinguished into a pair of partitions. It is formally defined as follows.

Definition 5 (Bipartite graph). A graph $G = (V, E)$ is said to be bipartite if its vertex set V can be partitioned into two nonempty and disjoint sets V_1 and V_2 such that $E = \{(u, v) | u \in V_1, v \in V_2\}$.

Therefore, a bipartite graph $G = (V, E)$ can also be represented as $G = (V_1, V_2, E)$. As the graphs may have subgraphs, bipartite graphs may also contain subgraphs. A biclique is a complete bipartite subgraph.

Definition 6 (Biclique). A bipartite subgraph $G = (V_1, V_2, E)$ is said to be a biclique if for each vertex pair $u \in V_1$ and $v \in V_2$, there is an edge (u, v) .

As can be seen, the edge set E of a biclique can be readily obtained from the two vertex sets V_1, V_2 , and therefore a biclique may be simply denoted as $G = (V_1, V_2)$.

Definition 7 (γ -quasi-biclique). In a bipartite graph $G = (V_1, V_2, E)$, a bipartite subgraph $G = (V'_1, V'_2, E')$, $V'_1 \subseteq V_1$, $V'_2 \subseteq V_2$, $E' \subseteq E$, is said to be a γ -quasi-biclique ($0 \leq \gamma \leq 1$) if the subgraph induced by these two sets of vertices contains at least $\lceil \gamma \cdot |V'_1| \cdot |V'_2| \rceil$ edges.